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The metal–insulator transition in n-type $In_{0.53}Ga_{0.47}As$

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Abstract. We present measurements of the low-temperature conductivity of a series of metallic n-type $In_{0.53}Ga_{0.47}As$ samples, having $k_F l < 3$. k_F is the Fermi wavenumber and l is the elastic scattering length. The data are extrapolated to zero temperature and the critical behaviour of the zero-temperature conductivity is examined. The temperature dependence of the conductivity appears consistent with terms due to electron–electron interactions and weak localisation. However, a detailed analysis utilising the magnetoconductivity reveals discrepancies with the theory, and it becomes clear that for these samples the corrections to the conductivity due to electron–electron interactions and weak localisation are *not* independent and additive. The inelastic scattering rate is inferred from the magnetoconductivity and its temperature dependence is found to change from one proportional to $T^{3/2}$ to one proportional to T as the metal–insulator transition is approached, in agreement with the theories of Isawa and Kaveh and co-workers.

1. Introduction

The critical behaviour of the zero-temperature conductivity, $\sigma(T = 0)$, at the metal–insulator transition (MIT), has received much attention in recent years. Mott (1972) predicted that $\sigma(T = 0)$ would drop discontinuously from a finite value denoted as σ_{min} to zero at the MIT, and that

$$\sigma_{min} \simeq 0.03e^2/\hbar a \tag{1}$$

where a is the average separation of scattering centres. Early experiments were interpreted as being consistent with that prediction. Abrahams *et al* (1979) applied one-parameter scaling theory to the disorder-induced MIT and predicted that $\sigma(T = 0)$ would drop discontinuously to zero at the MIT as the carrier concentration n was varied:

$$\sigma(T = 0) = \sigma_c(n/n_c - 1)^{\nu}. \tag{2}$$

σ_c is a constant predicted to be of the order of σ_{min}, n_c is the critical carrier concentration, and ν is a constant termed the critical exponent. In real systems electron–electron interactions are present and should be included in the theoretical description of MIT. A comprehensive description of the MIT including interactions and disorder does not yet exist, but attempts employing two-parameter scaling (McMillan 1981, Grest and Lee 1983) and field theory (Finkelshtein 1983, Castellani *et al* 1984) appear consistent with a continuous MIT given by equation (2). Mott (1984) has recalculated the behaviour of $\sigma(T = 0)$ and has confirmed equation (2) for zero applied magnetic field. However, Mott (1985) has suggested that the MIT is discontinuous in an applied magnetic field.

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For a disordered system without interactions ν has been predicted to be unity by many authors while for a disordered system with magnetic scattering $\nu = \frac{1}{2}$ has been predicted (Hikami 1981). Kaveh (1985) added interactions as a first-order peturbation to the disorder-induced MIT and predicted that for Hartree interaction $\nu = \frac{1}{2}$ and for exchange interaction $\nu = 1$. Experimentally, $\nu = 1$ has been observed for amorphous and compensated semiconductors in agreement with the prediction for a non-interacting system. For uncompensated Si and Ge $\nu = \frac{1}{2}$ has been observed, and it is not clear whether this corresponds to the case of magnetic scattering as suggested by Anderson (1985) or the case of Hartree interactions. Kaveh and Mott (1987) have suggested that the system must be many-valleyed and uncompensated to observe a critical exponent of $\frac{1}{2}$. For a comprehensive review of the critical exponent see Thomas (1985). The role of compensation is to both decrease the value of $k_F l$ at a given value of carrier concentration and tend to increase the critical exponent to unity.

To compare results with equation (2) and possibly obtain a value for ν, finite-temperature data must be extrapolated to $T = 0$ and, therefore, a functional form for the conductivity $\sigma(T)$ is required. Two temperature-dependent corrections to the conductivity have been calculated for the case of weak disorder, i.e. $k_F l \gg 1$ where k_F is the Fermi wavenumber and l is the elastic scattering length. Altshuler and Aronov (1985) have calculated the main contribution to the conductivity of a semiconductor sample due to electron–electron interactions to be

$$\Delta\sigma_i(T) = 0.915(e^3/2\pi^3\hbar^2)[(k_B/\hbar)(m^*/\sigma_B)k_F]^{1/2}\varphi(F)T^{1/2} \tag{3}$$

where

$$\varphi(F) = \tfrac{2}{3} - 8[(1 + F/2)^{3/2} - 1 - 3F/4]/F \tag{4}$$

$$F = (1/x^2)\ln(1 + x^2) \tag{5}$$

$$x = 2k_F/\kappa. \tag{6}$$

m^* is the electron effective mass, σ_B is the Boltzmann conductivity, F is the interaction parameter and κ is the inverse screening approximation. The second term is due to weak localisation or quantum interference and has been calculated by Kawabata (1981) and Berggren (1982) to be

$$\Delta\sigma_L(T) = (e^3/\pi^3\hbar^2)(m^*k_F/\sigma_B A)^{1/2}\, T^{p/2} \tag{7}$$

where

$$\tau_{in} = AT^{-p}. \tag{8}$$

τ_{in} is the inelastic scattering time which is expected to be related to electron–electron scattering at low temperatures. The exponent p is predicted to be 2 in the absence of disorder, $\frac{3}{2}$ in a diffusive system ($k_F l > 1$) and 1 in the critical limit ($k_F l \sim 1$) (Isawa 1984, Belitz and Wysokinski 1987).

Kaveh and Mott (1981) showed that for the case of weak disorder ($k_F l \gg 1$) the corrections due to interactions and weak localisation are independent and additive and, therefore, the conductivity has the form

$$\sigma(T) = a + bT^{1/2} + cT^{p/2} \qquad p = 2, \tfrac{3}{2}, 1. \tag{9}$$

b can be positive or negative, but c should be positive in the absence of spin–orbit scattering. The form of $\sigma(T)$ including interactions and weak localisation for the strong disorder near the MIT has not yet been calculated for the general case. Despite being

valid only for $k_F l \gg 1$, equation (9) has been fitted to data near the MIT to investigate the critical behaviour of $\sigma(T = 0)$—see, for example, Hirsch *et al* (1988), Mansfield *et al* (1985) and Katsumato *et al* (1987).

If equation (9) is fitted to a set of data, then the values of b and c obtained may be compared with equations (3) and (4) respectively, allowing experimental values of F and τ_{in} to be determined. The inelastic scattering time τ_{in} can also be determined from the low-field magnetoconductivity. Kawabata (1980) predicted a positive magnetoconductivity of the form

$$\Delta\sigma(B) = (e^2/2\pi^2\hbar)(eB/\hbar)^{1/2} f(\delta) \tag{10}$$

where

$$f(\delta) = \sum_{N=0}^{\infty} (2\sqrt{N+1+\delta} - \sqrt{N+\delta} - 1/\sqrt{N+\tfrac{1}{2}+\delta}) \tag{11}$$

and

$$\delta = \hbar/4ebB\tau_{in}D \tag{12}$$

due to the destruction of phase coherence that leads to weak localisation. B is the value of magnetic field and D is the diffusion constant. The magnetoconductivity due to electron–electron interactions is not expected to be significant unless $g\mu_B B > \pi k_B T$ (Isawa and Fukuyama 1984), where g is the Landé g-factor and μ_B is the Bohr magneton. By fitting equation (10) to the low-field magnetoconductivity data, a value for τ_{in} may be obtained.

The magnetoconductivity of weak localisation saturates at low field and the effects of weak localisation are completely eliminated for $g\mu_B B > \pi k_B T$ (Altshuler and Aronov 1985). Isawa and Fukuyama (1984) have predicted that at the point $g\mu_B B = \pi k_B T$ the slope of the $T^{1/2}$ correction to the conductivity changes. The function $\varphi(F)$ in equation (3) is replaced by

$$\varphi_B(F) = \tfrac{4}{3}[1 - (F/2 + 1)/(1 + \sqrt{1 + F/2})] \tag{13}$$

for applied magnetic fields such that $B > \pi k_B T/g\mu_B$. If a temperature dependence is measured in an applied magnetic field where the weak localisation is quenched, then experimental values of F may be obtained utilising either equations (3) and (4) or (3) and (13).

In the present work the conductivity of a series of $In_{0.53}Ga_{0.47}As$ samples ($k_F l < 3$) was measured as a function of temperature down to $\simeq$70 mK. The data are fitted to equation (9) and the fits are extrapolated to $T = 0$ to examine the critical behaviour of $\sigma(T = 0)$. The conductivity was also measured as a function of temperature at 0.3 T where the negative magnetoresistance due to weak localisation had saturated. Values of F are calculated from both $\sigma(T, B = 0)$ and $\sigma(T, B = 0.3\ \mathrm{T})$ and are compared. Equation (10) is fitted to the low-field magnetoresistance and the inelastic scattering rate obtained compared with the value determined from $\sigma(T, B = 0)$.

2. Critical behaviour of the zero-temperature conductivity

In table 1, the characteristics of the $In_{0.53}Ga_{0.47}As$ (InGaAs) samples used in this work are listed. The samples were grown nominally undoped by liquid-phase epitaxy, but residual Si and S caused the samples to be n-type. It was estimated that the compensation is $N_A/N_D \simeq \frac{1}{3}$ where N_A is the concentration of acceptors and N_D is the concentration of

Table 1. Characteristics of the InGaAs samples.

Sample	t (± 0.2 μm)	n (10^{15} cm^{-3})	$\sigma_{4.2K}$ (Ω^{-1} cm^{-1})	$k_F l$
BS374	2.25	12.8 ± 1.1	15.4 ± 1.4	2.60
BS396	3.3	11.6 ± 0.7	15.2 ± 0.9	2.64
BS405	4.3	10.8 ± 0.5	13.7 ± 0.6	2.44
BS395	3.3	8.1 ± 0.5	7.8 ± 0.5	1.54
BS379	2.6	6.4 ± 0.5	5.4 ± 0.4	1.14
BS390	3.6	5.5 ± 0.3	4.8 ± 0.3	1.06
BS393	3.75	3.4 ± 0.2	3.0 ± 0.2	0.78

donors. All of the samples were etched to a Hall bar geometry of overall dimensions 2.75×0.25 mm^2, and annealed AuNiGe alloy contacts were made. Sample thickness t was determined optically using a scanning electron microscope, and the surface depletion was subtracted utilising the method of Chandra *et al* (1979). The electron concentration $n = N_D - N_A$ was determined from the Hall constant which was found to be independent of temperature from 4.2 to 0.1 K. Assuming the conductivity at 4.2 K, $\sigma_{4.2K}$, to be the Boltzmann conductivity, values of $k_F l$ were estimated and found to be greater than unity for all but one of the samples. It was not possible to estimate N_D/N_A from a Brooks–Herring analysis because of the partial degeneracy of the samples at 77 K.

For each sample the resistance was measured using low-frequency phase-sensitive detection as a function of temperature in both a pumped ^{4}He cryostat and a dilution refrigerator, thus allowing the sample conductivity to be determined in the range 0.07–4.30 K. The uncertainty in a given temperature reading was less than 2%. All of the samples displayed a minimum in the conductivity near 1 K. The overall variation in conductivity was less than 10%, and typically 5%, for any given sample. For each sample equation (9) was fitted to the data for each predicted value of p. If equation (9) is fitted to the entire temperature range, then the resulting theoretical curves have the same general shape as the data for $p = 2$ or $\frac{3}{2}$, but the fit is not precise. Convincing fits are obtained to the data taken in the dilution refrigerator, i.e. for $T < 1$ K. For samples BS393, BS390 and BS379, $p = \frac{3}{2}$ gave the best fit, while for samples BS395, BS405 and BS396, $p = 2$ was most convincing. This was true regardless of the temperature range of the fit. The data taken below 1 K for BS374, the sample with the highest carrier concentration, fitted equation (9) with a very small negative value of c, so the best value of p is meaningless in this case. Fitting equation (9) to the entire temperature range of data indicates that $p = 2$ yields the best fit for BS374.

Figure 1 is a plot of conductivity versus $T^{1/2}$ for all seven samples. The full curves are the best fits of equation (9) to the data below 1 K. The interaction and weak-localisation terms in the fits will be examined closely in subsequent sections. At first glance the fits appear reasonable since to within the fitting error all yield $b < 0$ and $c \geqslant 0$, in accordance with the theory. It is also plausible that the value of p changes from 2 to $\frac{3}{2}$ as the disorder is increased (see § 1). Very close to the MIT, $p = 1$ is predicted.

The fits in figure 1 were extrapolated to zero temperature to obtain $\sigma(T = 1)$, i.e. $\sigma(T = 0) = a$. In figure 2 these values of $\sigma(T = 0)$ are plotted versus electron concentration with error bars reflecting both the fitting error and the uncertainty in sample thickness. The lowest experimental value of $\sigma(T = 0)$ is $\simeq 2.9\ \Omega^{-1}$ cm^{-1}, which is considerably larger than the minimum metallic conductivity (equation (1)), evaluated for InGaAs: $\sigma_{min} \simeq 0.9\ \Omega^{-1}$ cm^{-1}. Therefore the present data cannot conclusively determine whether the MIT is continuous or discontinuous. To be more conclusive InGaAs samples

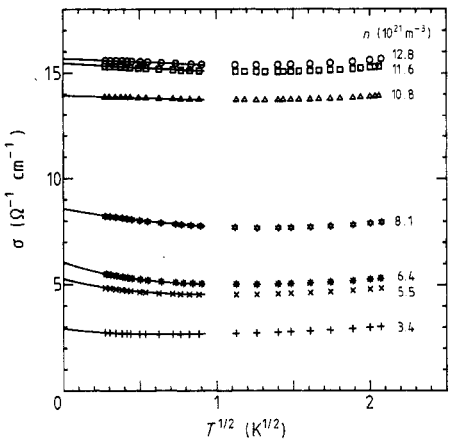

Figure 1. Conductivity versus $T^{1/2}$ for all seven InGaAs samples. The full curves are the best fits of equation (9) to the data taken below 1 K.

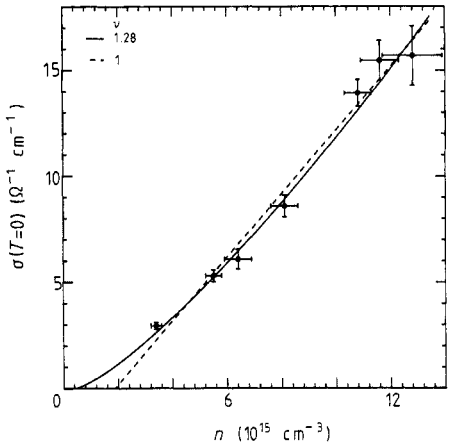

Figure 2. Zero-temperature conductivity versus electron concentration. The full curve is the best fit of equation (2) to the data, while the broken line is the best fit of equation (2) assuming $\nu = 1$.

with carrier concentrations $<3 \times 10^{15}$ cm^{-3} are required and such samples have proven to be extremely difficult to grow.

The full curve in figure 2 is the best fit of equation (2) to the data yielding $\sigma_c = 0.23 \pm 2.26\ \Omega^{-1}$ cm^{-1}, $n_c = (0.44 \pm 2.72) \times 10^{15}$ cm^{-3} and $\nu = 1.28 \pm 0.48$. The value of ν obtained is close to the value unity, which is predicted by theories that include only the effect of disorder at the MIT. It is possible that disorder is stronger in InGaAs because of the lattice disorder in the positions of the In and Ga that leads to alloy scattering. Also, the present samples are compensated. The existence of negative $T^{1/2}$ correction to $\sigma(T)$ indicates that electron–electron interactions are present and that the Hartree term is larger than the exchange term. Kaveh (1985) predicts $\nu = \frac{1}{2}$ for a disordered system with Hartree interactions, which appears inconsistent with the data. Experimentally $\nu \sim 1$ has been observed for many compensated and amorphous semiconductors, in agreement with the present results.

Although the value of $\sigma_c = 0.23 \pm 2.26\ \Omega^{-1}$ cm^{-1} obtained is equal to $\sigma_{min} = 0.9\ \Omega^{-1}$ cm^{-1} to within the fitting error, it is hardly conclusive. Similarly, $n_c = (0.44 \pm 2.72) \times 10^{15}$ cm^{-3} agrees with the value predicted by the Mott criterion (Mott 1961), 2.9×10^{15} cm^{-3}, although there is a large uncertainty. If the critical exponent is fixed at unity, fitting to the scaling theory yields $\sigma_c = 2.8 \pm 0.8\ \Omega^{-1}$ cm^{-1} and $n_c = (1.8 \pm 0.4) \pm 10^{15}$ cm^{-3}, and the corresponding fit is shown as a broken line in figure 2.

3. The interaction correction to the conductivity

By equating equation (3) with the experimentally determined correction to σ, experimental values of F were obtained for each sample. In figure 3 these values of F are plotted, as squares, versus carrier concentration with the theoretical prediction, equation (5), included as a full line. The errors arise from the uncertainty in thickness and the uncertainty in the fitting procedure. The broken lines have been added to guide the eye. All of the experimentally determined values of F are greater than unity, even though F is theoretically restricted to the range 0 to 1. Despite the error, F is apparently a strong

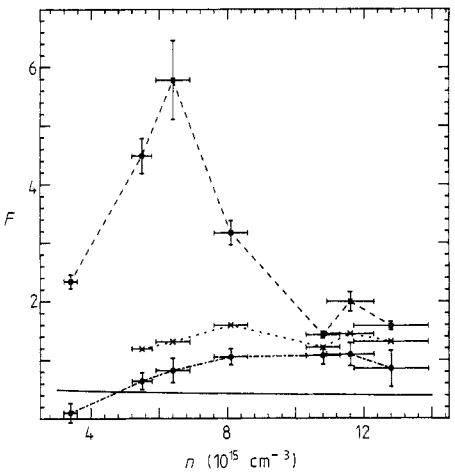

Figure 3. The interaction parameter versus electron concentration. The values represented as squares, crosses and circles were calculated from the fits to $\sigma(T, B = 0)$, $\sigma(T > T_B, B = 0.3\,\mathrm{T})$ and $\sigma(T < T_B, B = 0.3\,\mathrm{T})$ respectively, where $T_B = g\mu_B B/\pi k_B$. The full line represents the theoretical prediction, while the various broken lines serve as guides to the eye.

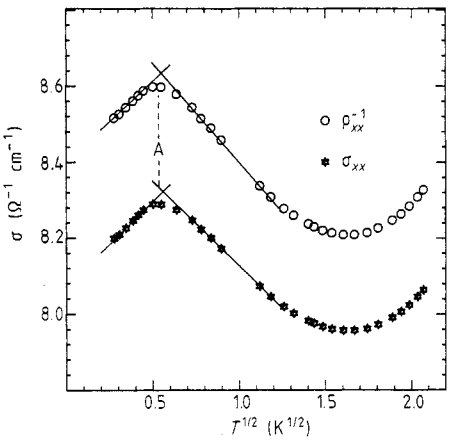

Figure 4. σ_{xx} and $\rho^{-1}{}_{xx}$ versus $T^{1/2}$ for sample BS395 at $B = 0.3$ T. Both curves are linear against $T^{1/2}$ below 1 K with a sharp change in slope at the point of full spin splitting (point A), $g\mu_B B = \pi k_B T$. The full lines are the best linear fits to the two $T^{1/2}$ regions.

function of n with a peak value of $\simeq 6$ near $n = 6 \times 10^{15}$ cm^{-3}, which roughly corresponds to $k_F l \simeq 1$. It is possible that for samples with a higher carrier concentration, experimentally determined values of F would approach the predictions of theory. The present results clearly disagree with the theory.

Large values of F ($F > 1$) have been experimentally determined for many systems. In particular Long and Pepper (1984) determined $F \simeq 2.3$ for a bulk metallic InP sample, and Morita *et al* (1984a) measured values greater than unity for a series of GaAs samples with peak value of $F \simeq 2.5$ for a sample with $k_F l \simeq 1$.

The InGaAs samples exhibited a low-field negative magnetoresistance due to weak localisation that saturated by 0.3 T at temperatures below 1 K. The Landé g-factor for InGaAs is 4.5 and for an applied magnetic field of 0.3 T the condition $g\mu_B B = \pi k_B T$ is satisfied at a temperature of $T_B = 0.29$ K. Above 0.29 K the temperature dependence of the conductivity should be given by equation (3) with $\varphi(F)$ specified by equation (4), while below 0.29 K it should be given by equation (3) with $\varphi(F)$ replaced by $\varphi_B(F)$ as defined in equation (13).

In the interaction theory the conductivity is implicitly assumed to be isotropic, but because of the low carrier concentration of the InGaAs samples, the off-diagonal elements in the resistivity tensor become significant at low fields. It is not obvious how best to compare the data with the theory in this case. Taking the magnetic field to be in the z direction, the conductivity perpendicular to the field is given by $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)$. That quantity will include a magnetoconductivity due to the magnetic field dependence of ρ_{xv} which is not included in the theoretical magnetoconductivity since ρ_{xy} is assumed to be zero. The quantity ρ_{xx}^{-1} will not include that additional magnetoconductivity but it is not a true conductivity if ρ_{xy} is finite.

Employing a four-terminal low-frequency circuit, ρ_{xx} and ρ_{xy} were determined for each InGaAs sample as a function of temperature over the range 0.07–4.2 K in an applied

magnetic field of 0.3 T. In figure 4, σ_{xx} and ρ_{xx}^{-1} are plotted versus $T^{1/2}$ for sample BS395, and it is readily apparent that although they differ in absolute magnitude, σ_{xx} and ρ_{xx}^{-1} show the same temperature dependences in this case. It is not crucial which quantity is used for the comparison with theory and in this instance σ_{xx} was used since it is a true conductivity. As illustrated for sample BS395 in figure 4, the conductivities of the InGaAs samples do indeed have a $T^{1/2}$ correction below 1 K that changes slope near 0.29 K where $g\mu_B B = \pi k_B T$ is satisfied. It appears that above 1 K quantum interference also contributes to the conductivity. For sample BS393, doped closest to the MIT, the conductivity does not exhibit a clear $T^{1/2}$ correction above 0.29 K, but every other sample distinctly displays the two $T^{1/2}$ regions.

For each sample the interaction parameter F was determined from the slope of the $T^{1/2}$ correction above and below $T_B = 0.29$ K, and these values are plotted in figure 3 as crosses and full circles respectively. The values determined at 0.3 T are much smaller than the values obtained at zero field, and are relatively insensitive to n, lying on average close to unity. The points derived from data below T_B lie consistently below those derived from data above T_B. The broken lines are intended as guides to the eye. The uncertainty in the values of F determined from the data taken above T_B at 0.3 T is too small to show on the scale of the diagram.

In the theory of electron–electron interactions the correction to the conductivity due to interactions in the particle–particle channel (the Cooper channel) is assumed to be negligible for semiconductors. This correction is not quenched by an applied magnetic field until $g\mu_B B > \pi k_B T$ is satisfied. It is therefore conceivable that Cooper channel interactions are responsible for the difference between the values of F calculated from the data below T_B and those calculated from the data above T_B. However, an estimate of the correction to σ due to Cooper channel effects based on the calculations of Isawa *et al* (1982) indicates that the correction is indeed negligible. It may also be possible that a small amount of weak localisation remains at 0.3 T, even though significant negative magnetoresistance is no longer observed, and can account for some of the difference between the two values of F derived from the 0.3 T data. For $g\mu_B B > \pi k_B T$, weak localisation is entirely suppressed.

The large difference between the values of F obtained from the zero-field data and the values obtained from the 0.3 T data cannot be explained by the effects of the Cooper channel or a small amount of weak localisation at 0.3 T. It appears that the quantum interference correction and electron–electron interaction correction to the conductivity cannot simply be added for these samples, but that the interaction term is enhanced by the presence of quantum interference resulting in the determination of very large values of the interaction parameter. This enhancement peaks at $k_F l \simeq 1$. It must be noted that even when quantum interference is quenched by an applied magnetic field, the theory of electron–electron interactions still yields only qualitative agreement with the observed behaviour. Perhaps the theories derived for the regime of weak disorder ($k_F l \gg 1$) are simply not valid for these samples.

4. The inelastic scattering rate

Newson *et al* (1985) made a detailed investigation of the inelastic scattering rate in similar InGaAs samples in the temperature range 1.3–4.2 K. By fitting equation (10) to the observed low-field magnetoresistance, they found that the inelastic scattering rate followed $\tau_{in}^{-1} = aT^{3/2} + b$ for all of the samples ($1 < k_F l < 10$). The first term was ascribed to electron–electron scattering while the second term was attributed to scattering from compositional inhomogeneities. They also demonstrated that magnetic and spin–orbit scattering were not present, so equation (10) may be considered valid.

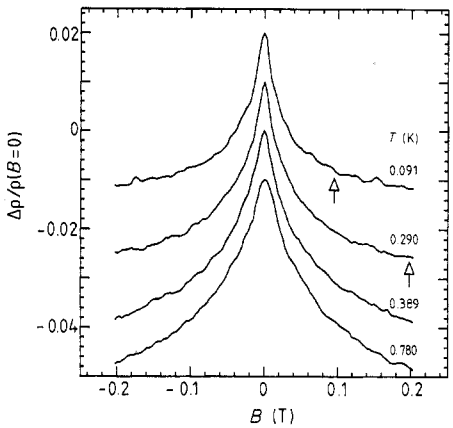

Figure 5. $\Delta\rho/\rho(B=0)$ versus B for sample BS374 at 0.091, 0.290, 0.389 and 0.780 K. The large arrows mark the point where $g\mu_B B = \pi k_B T$ is satisfied.

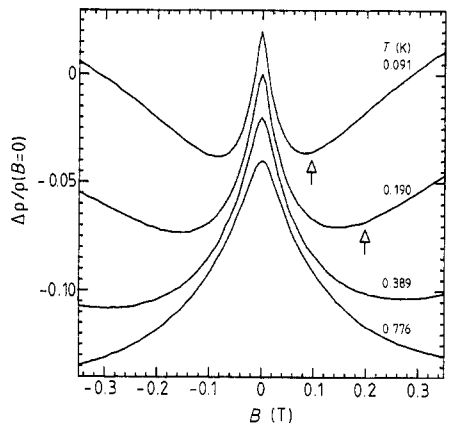

Figure 6. $\Delta\rho/\rho(B=0)$ versus B for sample BS379 at 0.091, 0.190, 0.389 and 0.776 K. Positive magnetoresistance is seen at fields above the point corresponding to $g\mu_B B = \pi k_B T$ which is marked with an arrow.

The present samples also display a low-field negative magnetoresistance. The magnetoresistance of two samples, BS374 and BS379, was investigated in detail to see if the inelastic rate determined from the magnetoresistance is consistent with the weak-localisation correction to $\sigma(T)$. Following the work of Newson *et al*(1985) it may be shown that magnetic and spin–orbit scattering are not significant in these samples. Therefore, as long as the magnetoresistance due to interactions is insignificant equation (10) may be applied.

The low-field magnetoresistances of samples BS374 and BS379 were measured at over 10 temperatures in the range 0.07 to 0.8 K. Figure 5 is a plot of $\Delta\rho/\rho(B=0)$ versus B for BS374 ($k_F l \simeq 2.60$) at sample temperatures of 0.091, 0.290, 0.389 and 0.780 K, where some of the curves have been offset for clarity. The points corresponding to $g\mu_B B = \pi k_B T$ are marked with arrows, but no positive magnetoresistance due to interactions can be seen. The overall size of the magnetoresistance is fairly constant with temperature, about 3% at 0.1 T, but the peak at $B = 0$ gets sharper as the temperature is reduced.

Figure 6 is a similar plot of $\Delta\rho/\rho(B=0)$ versus B at four different temperatures for sample BS379 ($k_F l \simeq 1.14$) and several differences from the data for BS374 are obvious. The overall magnitude of the magnetoresistance is larger, about 7% at 0.1 T, and at the point where $g\mu_B B = \pi k_B T$ is satisfied, marked by an arrow, a positive magnetoresistance due to interactions enters. The peak at $B = 0$ is broader, but it does get sharper as the temperature is reduced, just as it does for BS374.

A simple computer program was used to fit equation (10) to the experimental magnetoresistance data, which were digitised, averaged over both field directions, and converted to magnetoconductivity data. At the low fields for which reasonable fits could be obtained, ρ_{xy} was negligible and was therefore ignored. Good fits to the data for sample BS374 were obtained if the magnetic field range was restricted to $B \leqslant 0.01$ T and for sample BS379 if the field was restricted to $B \leqslant 0.015$ T. The limited range of the fits may be due to the samples not lying well within the theory's range of validity ($k_F l \geqslant 1$). It should be noted that in the limit of weak fields, equation (10) reduces to $\Delta\sigma \propto B^2$ with the slope proportional to $\tau_{in}^{3/2}$, and recent work shows that a similar law applies to the

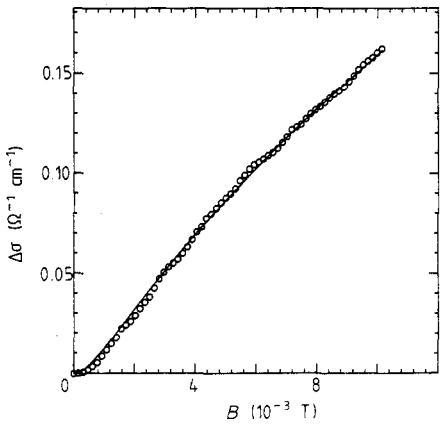

Figure 7. $\Delta\sigma = \sigma(B) - \sigma(0)$ versus B for sample BS374 at 0.293 K. The full curve is the best fit of equation (10) to the data points up to 0.01 T.

low-field magnetoresistance in the hopping regime (Sivan *et al* 1988). Previous work on metallic n-type InGaAs (Newson *et al* 1985) and GaAs (Morita *et al* 1984b) has used only the B^2 region to determine τ_{in}^{-1}.

Equation (10) was fitted to the magnetoconductivity data for BS374 and BS379 up to fields of 0.01 and 0.015 T respectively, for several temperatures below 1 K. A typical fit for sample BS374 is shown in figure 7, and it can be seen that it extends beyond the B^2 region. From each fit the inelastic scattering time was obtained, and the inelastic scattering rate may therefore be examined as a function of temperature. For sample BS374 the data clearly follow $\tau_{in}^{-1} \propto T^{3/2}$, as shown in figure 8, which has been predicted for electron–electron scattering in the diffusive limit (Schmid 1974, Isawa 1984). The magnitude of the scattering rate agrees well with the results of Newson *et al* (1985), but the present results do not include a temperature-independent term, possibly because the present samples are more homogeneous and the scattering from compositional inhomogeneities is weak. For sample BS379 the data clearly follows $\tau_{in}^{-1} \propto T$, as shown in figure 9, which has been predicted for electron–electron scattering in the critical limit (Isawa 1984, Altshuler and Aronov 1985). Kaveh *et al* (1987) show that this temperature

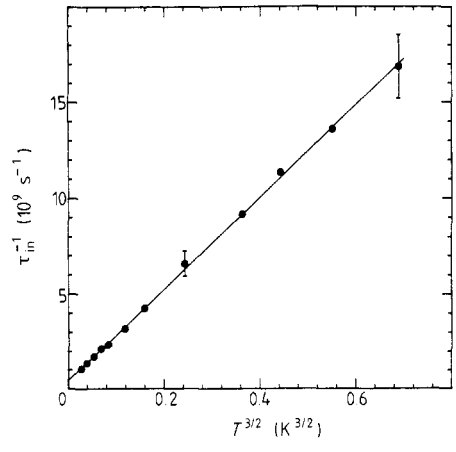

Figure 8. τ_{in}^{-1} versus $T^{3/2}$ for sample BS374. The sample error bars represent a 10% uncertainty in the values of τ_{in}, and the full line is a linear regression of the data.

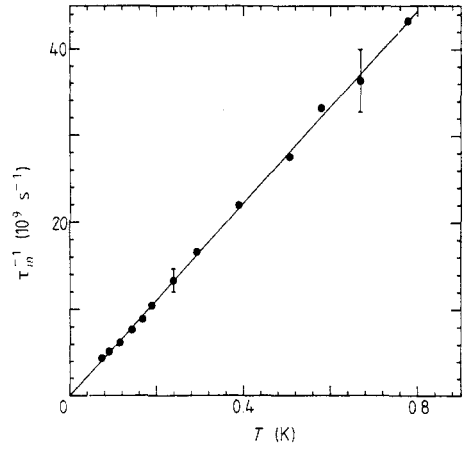

Figure 9. τ_{in}^{-1} versus T for sample BS379. The sample error bars represent 10% uncertainty in the values of τ_{in}, and the full line is a linear regression of the data.

dependence is due to the inelastic length being the shortest at the MIT and they present supporting results for GaAs. The present result also agrees well with the scattering rate measured by Morita *et al* (1984b) for a GaAs sample with $k_F l \simeq 1.8$. Error bars representing an estimated 10% uncertainty in τ_{in} are displayed in figures 8 and 9 and it is seen that the data are clearly linear to within that error.

When these results are compared with the temperature dependences of the conductivity completed in zero field the outcome is very surprising. For sample BS374 the zero-field conductivity followed $\sigma(T) = (15.7 \pm 1.4) - (0.32 \pm 0.05)T^{1/2} - (0.04 \pm 0.04)T\,\Omega^{-1}\,\mathrm{cm}^{-1}$ while the measured inelastic scattering rate implies a weak-localisation correction to the conductivity of $\simeq 0.3T^{3/2}\,\Omega^{-1}\,\mathrm{cm}^{-1}$. Despite the interaction term in $\sigma(T)$ no evidence of interactions is present in the magnetoresistance. Furthermore, samples having a similar carrier concentration exhibited temperature-dependent corrections proportional to T that were attributed to weak localisation. For sample BS379 the zero-field conductivity followed $\sigma(T) = (6.1 \pm 0.5) - (3.2 \pm 0.2)T^{1/2} + (2.1 \pm 0.2)T^{3/4}\,\Omega^{-1}\,\mathrm{cm}^{-1}$, while the measured inelastic scattering rate implies a weak-localisation correction to the conductivity of $\simeq 1.8T^{1/2}\,\Omega^{-1}\,\mathrm{cm}^{-1}$. The agreement is poor; even the temperature dependence of the weak-localisation correction is wrong.

5. Conclusions

The low-temperature conductivity of a series of metallic InGaAs samples was found to follow a function that appeared qualitatively consistent with corrections to the conductivity due to electron–electron interactions and weak localisation. The fits were extrapolated to zero temperature and the zero-temperature conductivity was found to be consistent with scaling theory and a critical exponent of 1.28 ± 0.48. However, samples close enough to the MIT to rule out a discontinuous transition unequivocally were not available.

A detailed investigation of the two temperature-dependent corrections to the conductivity revealed that they do not agree with the predictions of theory. In particular the term ascribed to interactions was much larger than predicted in zero field and the term attributed to weak localisation had the wrong temperature dependence when compared with the inelastic scattering rate. It is clear that for these samples ($k_F l < 3$), the interaction and localisation corrections to the conductivity are not independent and additive as is generally assumed for systems with $k_F l > 1$. All of the theory derived for weakly-disordered systems ($k_F l \gg 1$) might have a more limited range of applicability (for example $k_F l > 5$) than is widely assumed ($k_F l \geqslant 1$).

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